Chapter 13: Curve Sketching

13.1: Relative Extrema (Q. 1—71)

In Problems 5–8, the derivative of a differentiable function f is given. Find the open intervals on which f is **(a)** *increasing;* **(b)** *decreasing; and* **(c)** *find the x-values of all relative extrema.*

5. $f'(x) = (x+3)(x-1)(x-2)$

6. $f'(x) = x^2(x-2)^3$

7. $f'(x) = (x+1)(x-3)^2$

8. $f'(x) = \dfrac{x(x+2)}{x^2+1}$

In Problems 9–52, determine where the function is **(a)** *increasing;* **(b)** *decreasing; and* **(c)** *determine where relative extrema occur. Do not sketch the graph.*

9. $y = -x^3 - 1$

10. $y = x^2 + 4x + 3$

11. $y = 5 - 2x - x^2$

12. $y = x^3 - \frac{5}{2}x^2 - 2x + 6$

13. $y = -\frac{x^3}{3} - 2x^2 + 5x - 2$

14. $y = -\frac{x^4}{4} - x^3$

15. $y = x^4 - 2x^2$

16. $y = x^3 - \frac{3}{2}x^2 - 36x$

17. $y = x^3 - \frac{7}{2}x^2 + 2x - 5$

18. $y = x^3 - 6x^2 + 12x - 6$

19. $y = 2x^3 - \frac{19}{2}x^2 + 10x + 2$

20. $y = -5x^3 + x^2 + x - 7$

21. $y = 1 - 3x + 3x^2 - x^3$

22. $y = \frac{9}{5}x^5 - \frac{47}{3}x^3 + 10x$

23. $y = 3x^5 - 5x^3$

24. $y = 3x - \frac{x^6}{2}$ (Remark: $x^4 + x^3 + x^2 + x + 1 = 0$ has no real roots.)

25. $y = -x^5 - 5x^4 + 200$

26. $y = \frac{x^4}{4} - \frac{5x^3}{3} + \frac{7x^2}{2} - 3x$

27. $y = 8x^4 - x^8$

28. $y = \frac{4}{5}x^5 - \frac{13}{3}x^3 + 3x + 4$

29. $y = (x^2 - 4)^4$

30. $y = \sqrt[3]{x}(x-2)$

31. $y = \dfrac{3}{x+2}$

32. $y = \dfrac{3}{x}$

33. $y = \dfrac{10}{\sqrt{x}}$

34. $y = \dfrac{ax+b}{cx+d}$
(a) for $ad - bc > 0$
(b) for $ad - bc < 0$

35. $y = \dfrac{x^2}{2-x}$

36. $y = \dfrac{27x^2}{2} + \dfrac{1}{x}$

37. $y = \dfrac{x^2-3}{x+2}$

38. $y = \dfrac{2x^2}{4x^2-25}$

39. $y = \dfrac{ax^2+b}{cx^2+d}$ for $d/c < 0$
(a) for $ad - bc > 0$
(b) for $ad - bc < 0$

40. $y = \sqrt[3]{x^3 - 9x}$

41. $y = (x+1)^{2/3}$

42. $y = x^2(x+3)^4$

43. $y = x^3(x-6)^4$

44. $y = (1-x)^{2/3}$

45. $y = e^{-\pi x} + \pi$

46. $y = x^3 \ln x$

47. $y = x^2 - 9\ln x$

48. $y = x^{-1}e^x$

49. $y = e^x - e^{-x}$

50. $y = e^{-x^2/2}$

51. $y = x^2 \ln x$

52. $y = (x^2+1)e^{-x}$

In Problems 53–64, determine intervals on which the function is increasing; intervals on which the function is decreasing; relative extrema; symmetry; and those intercepts that can be obtained conveniently. Then sketch the graph.

53. $y = x^2 - 3x - 10$

54. $y = 2x^2 + x - 10$

55. $y = 3x - x^3$

56. $y = x^4 - 81$

57. $y = 2x^3 - 9x^2 + 12x$

58. $y = 2x^3 - x^2 - 4x + 4$

59. $y = x^4 - 2x^2$

60. $y = x^6 - \frac{6}{5}x^5$

61. $y = (x-1)^3(x-2)^2$

62. $y = \sqrt{x}(x^2 - x - 2)$

63. $y = 2\sqrt{x} - x$

64. $y = x^{5/3} - 2x^{2/3}$

65. Sketch the graph of a continuous function f such that $f(2) = 2$, $f(4) = 6$, $f'(2) = f'(4) = 0$, $f'(x) < 0$ for $x < 2$, $f'(x) > 0$ for $2 < x < 4$, f has a relative maximum at 4, and $\lim_{x\to\infty} f(x) = 0$.

66. Sketch the graph of a continuous function f such that $f(0) = 0, f(1) = 1, f(2) = 2, f(3) = 1, f'(0) = 0 = f'(2)$, there is a vertical tangent line when $x = 1$ and when $x = 3, f'(x) < 0$ for x in $(-\infty, 0)$ and x in $(2, 3), f'(x) > 0$ for x in $(0, 1)$ and x in $(1, 2)$ and x in $(3, \infty)$.

67. Average Cost If $c_f = 25{,}000$ is a fixed-cost function, show that the average fixed-cost function $\bar{c}f = c_f/q$ is a decreasing function for $q > 0$. Thus, as output q increases, each unit's portion of fixed cost declines.

68. Marginal Cost If $c = 3q - 3q^2 + q^3$ is a cost function, when is marginal cost increasing?

69. Marginal Revenue Given the demand function

$$p = 500 - 5q$$

find when marginal revenue is increasing.

70. Cost Function For the cost function $c = \sqrt{q}$, show that marginal and average costs are always decreasing for $q > 0$.

71. Revenue For a manufacturer's product, the revenue function is given by $r = 180q + 87q^2 - 2q^3$. Determine the output for maximum revenue.

13.3: Concavity (Q. 1—19, 23, 25, 29—55, 61—67)

PROBLEMS 13.3

In Problems 1–6, a function and its second derivative are given. Determine the concavity of f and find x-values where points of inflection occur.

1. $f(x) = x^4 - 3x^3 - 6x^2 + 6x + 1; f''(x) = 6(2x+1)(x-2)$

2. $f(x) = \frac{x^5}{20} + \frac{x^4}{4} - 2x^2; f''(x) = (x-1)(x+2)^2$

3. $f(x) = \frac{x^2+3x+1}{x^2+2x+1}; f''(x) = \frac{2x-4}{(x+1)^4}$

4. $f(x) = \frac{x^2}{(x-1)^2}; f''(x) = \frac{2(2x+1)}{(x-1)^4}$

5. $f(x) = \frac{x^2+1}{x^2-2}; f''(x) = \frac{6(3x^2+2)}{(x^2-2)^3}$

6. $f(x) = x\sqrt{a^2-x^2}; f''(x) = \frac{x(2x^2-3a^2)}{(a^2-x^2)^{3/2}}$

In Problems 7–34, determine concavity and the x-values where points of inflection occur. Do not sketch the graphs.

7. $y = -2x^2 + 4x$

8. $y = 4x^2 - 375x + 947$

9. $y = 4x^3 + 12x^2 - 12x$

10. $y = x^3 - 6x^2 + 9x + 1$

11. $y = ax^3 + bx^2 + cx + d$

12. $y = x^4 - 8x^2 - 6$

13. $y = x^5 - 10x^4 + \frac{110}{3}x^3 - 60x^2$

14. $y = -\frac{x^4}{4} + \frac{9x^2}{2} + 2x$

15. $y = 2x^{1/5}$

16. $y = \frac{a}{x^3}$

17. $y = \frac{x^4}{2} + \frac{19x^3}{6} - \frac{7x^2}{2} + x + 5$

18. $y = \frac{2}{4}x^4 + \frac{11}{6}x^3 + \frac{3}{2}x^2 + \frac{7}{5}x + \frac{3}{5}$

19. $y = \frac{1}{20}x^5 - \frac{1}{4}x^4 + \frac{1}{6}x^3 - \frac{1}{2}x - \frac{2}{3}$

20. $y = \frac{1}{10}x^5 - 3x^3 + 17x + 43$

21. $y = \frac{1}{30}x^6 - \frac{7}{12}x^4 + 6x^2 + 5x - 4$

22. $y = x^6 - 3x^4$

23. $y = \frac{x-1}{x+1}$

24. $y = 1 - \frac{1}{x^2}$

25. $y = \frac{x^2}{x^2+1}$

26. $y = \frac{ax^2}{x+b}$

27. $y = \frac{21x+40}{6(x+3)^2}$

28. $y = (x^2-12)^2$

29. $y = 5e^x$

30. $y = e^x - e^{-x}$

31. $y = axe^x$

32. $y = xe^{x^2}$

33. $y = \ln x$

34. $y = \frac{x^2+1}{3e^x}$

In Problems 35–62, determine intervals on which the function is increasing, decreasing, concave up, and concave down; relative maxima and minima; inflection points; symmetry; and those intercepts that can be obtained conveniently. Then sketch the graph.

35. $y = x^2 - x - 6$

36. $y = x^2 + a$ for $a > 0$

37. $y = 5x - 2x^2$

38. $y = -1 - x^2 + 2x$

39. $y = x^3 - 9x^2 + 24x - 19$

40. $y = x^3 - 25x^2$

41. $y = \frac{x^3}{3} - 5x$

42. $y = x^3 - 6x^2 + 9x$

43. $y = x^3 + 3x^2 + 3x + 1$

44. $y = 2x^3 + \frac{5}{2}x^2 + 2x$

45. $y = 4x^3 - 3x^4$

46. $y = -x^3 + 8x^2 - 5x + 3$

47. $y = -2 + 12x - x^3$

48. $y = -(3x+2)^3$

49. $y = 2x^3 - 6x^2 + 6x - 2$

50. $y = \frac{x^5}{100} - \frac{x^4}{20}$

51. $y = 16x - x^5$

52. $y = x^2(x-1)^2$

53. $y = 6x^4 - 8x^3 + 3$

54. $y = 3x^5 - 5x^3$

55. $y = 4x^2 - x^4$

56. $y = x^2e^x$

57. $y = x^{1/3}(x-8)$

58. $y = (x+1)^2(x-2)^2$

59. $y = 4x^{1/3} + x^{4/3}$

60. $y = (x+1)\sqrt{x+4}$

61. $y = 2x^{2/3} - x$

62. $y = 5x^{2/3} - x^{5/3}$

63. Sketch the graph of a continuous function f such that $f(0) = 0 = f(3), f'(1) = 0 = f'(3), f''(x) < 0$ for $x < 2$, and $f''(x) > 0$ for $x > 2$.

64. Sketch the graph of a continuous function f such that $f(4) = 4, f'(4) = 0, f''(x) < 0$ for $x < 4$, and $f''(x) > 0$ for $x > 4$.

65. Sketch the graph of a continuous function f such that $f(1) = 1, f'(1) = 0$, and $f''(x) < 0$ for all x.

66. Sketch the graph of a continuous function f such that $f(1) = 1$, both $f'(x) < 0$ and $f''(x) < 0$ for $x < 1$, and both $f(x) > 0$ and $f''(x) < 0$ for $x > 1$.

67. Demand Equation Show that the graph of the demand equation $p = \frac{100}{q+2}$ is decreasing and concave up for $q > 0$.

68. Average Cost For the cost function

$$c = q^2 + 3q + 2$$

show that the graph of the average-cost function $\bar{c}$ is concave up for all $q > 0$.

13.5: Asymptotes: (Q. 1—49, 53)

PROBLEMS 13.5

In Problems 1–24, find the vertical asymptotes and the nonvertical asymptotes for the graphs of the functions. Do not sketch the graphs.

1. $y = \frac{x}{x-1}$

2. $y = \frac{x+1}{x}$

3. $f(x) = \frac{x+5}{2x+7}$

4. $y = \frac{2x+1}{2x+1}$

5. $y = \frac{3}{x^3}$

6. $y = 1 - \frac{2}{x^2}$

7. $y = \frac{1}{x^2-1}$

8. $y = \frac{x}{x^2-9}$

9. $y = x^2 - 5x + 5$

10. $y = \frac{x^3}{x^2-1}$

11. $f(x) = \frac{2x^2}{x^2+x-6}$

12. $f(x) = \frac{x^3}{5}$

13. $y = \frac{15x^2+31x+1}{x^2-7}$

14. $y = \frac{2x^3+1}{3x(2x-1)(4x-3)}$

15. $y = \frac{3}{x-5} + 7$

16. $f(x) = \frac{x^2-1}{2x^2-9x+4}$

17. $f(x) = \frac{3-x^4}{x^3+x^2}$

18. $y = \frac{5x^2+7x^3+9x^4}{3x^2}$

19. $y = \frac{x^2-3x-4}{1+4x+4x^2}$

20. $y = \frac{x^3+1}{1-x^3}$

21. $y = \frac{9x^2-16}{2(3x+4)^2}$

22. $y = \frac{2}{5} + \frac{2x}{12x^2+5x-2}$

23. $y = 5e^{x-3} - 2$

24. $f(x) = 12e^{-x}$

In Problems 25–46, determine intervals on which the function is increasing, decreasing, concave up, and concave down; relative maxima and minima; inflection points; symmetry; vertical and nonvertical asymptotes; and those intercepts that can be obtained conveniently. Then sketch the curve.

25. $y = \frac{1}{x^3}$

26. $y = \frac{2}{2x-3}$

27. $y = \frac{x}{x-1}$

28. $y = \frac{50}{\sqrt{3x}}$

29. $y = x^2 + \frac{1}{x^2}$

30. $y = \frac{x^2+x+1}{x-2}$

31. $y = \frac{1}{x^2-1}$

32. $y = \frac{1}{x^2+1}$

33. $y = \frac{2+x}{3-x}$

34. $y = \frac{1+x}{x^2}$

35. $y = \frac{x^2}{x-1}$

36. $y = \frac{x^3+1}{x}$

37. $y = \frac{9}{9x^2-6x-8}$

38. $y = \frac{4x^2+2x+1}{2x^2}$

39. $y = \frac{3x+1}{(3x-2)^2}$

40. $y = \frac{3x+5}{(7x+11)^2}$

41. $y = \frac{x^2-1}{x^3}$

42. $y = \frac{3x}{(x-2)^2}$

43. $y = 2x + 1 + \frac{1}{x-1}$

44. $y = \frac{3x^4+1}{x^3}$

45. $y = \frac{1-x^2}{x^2-1}$

46. $y = 3x + 2 + \frac{1}{3x+2}$

47. Sketch the graph of a function f such that $f(0) = 0$, there is a horizontal asymptote $y = 1$ for $x \to \pm\infty$, there is a vertical asymptote $x = 2$, both $f'(x) < 0$ and $f''(x) < 0$ for $x < 2$, and both $f'(x) < 0$ and $f''(x) > 0$ for $x > 2$.

48. Sketch the graph of a function f such that $f(0) = -4$ and $f(4) = -2$, there is a horizontal asymptote $y = -3$ for $x \to \pm\infty$, there is a vertical asymptote $x = 2$, both $f'(x) < 0$ and $f''(x) < 0$ for $x < 2$, and both $f'(x) < 0$ and $f''(x) > 0$ for $x > 2$.

49. Sketch the graph of a function f such that $f(0) = 0$, there is a horizontal asymptote $y = 0$ for $x \to \pm\infty$, there are vertical asymptotes $x = -1$ and $x = 2$, $f'(x) < 0$ for $x < -1$ and $-1 < x < 2$, and $f''(x) < 0$ for $x > 2$.

50. Sketch the graph of a function f such that $f(0) = 0$, there are vertical asymptotes $x = -1$ and $x = 1$, there is a horizontal asymptote $y = 0$ for $x \to \pm\infty$. $f'(x) < 0$ for x in $(-\infty, -1)$, in $(-1, 1)$, and in $(1, \infty)$. $f''(0) = 0$; $f''(x) < 0$ for x in $(-\infty, -1)$ and in $(0, 1)$; $f''(x) > 0$ for x in $(-1, 0)$ and in $(1, \infty)$.

51. Purchasing Power In discussing the time pattern of purchasing, Mantell and Sing[11] use the curve

$$y = \frac{x}{a+bx}$$

as a mathematical model. Find the asymptotes for their model.

52. Sketch the graphs of $y = 6 - 3e^{-x}$ and $y = 6 + 3e^{-x}$. Show that they are asymptotic to the same line. What is the equation of this line?

53. Market for Product For a new product, the yearly number of thousand packages sold, y, t years after its introduction is predicted to be given by

$$y = f(t) = 250 - 83e^{-t}$$

Show that $y = 250$ is a horizontal asymptote for the graph. This reveals that after the product is established with consumers, the market tends to be constant.

54. Graph $y = \frac{x^2-2}{x^3 + \frac{7}{2}x^2 + 12x + 1}$. From the graph, locate any horizontal or vertical asymptotes.

55. With a graphing utility, graph $y = \frac{2x^3-2x^2+6x-1}{x^3-6x^2+11x-6}$. From the graph, locate any horizontal or vertical asymptotes.

56. Graph $y = \frac{\ln(x+4)}{x^2-8x+5}$ in the standard window. The graph suggests that there are two vertical asymptotes of the form $x = k$, where $k > 0$. Also, it appears that the graph "begins" near $x = -4$. As $x \to -4^+$, $\ln(x+4) \to -\infty$ and $x^2 - 8x + 5 \to 53$. Thus, $\lim_{x \to -4^+} y = -\infty$. This gives the vertical asymptote $x = -4$. So, in reality, there are *three* vertical asymptotes. Use the zoom feature to make the asymptote $x = -4$ apparent from the display.

57. Graph $y = \frac{0.34e^{0.7x}}{4.2 + 0.71e^{0.7x}}$, where $x > 0$. From the graph, determine an equation of the horizontal asymptote by examining the y-values as $x \to \infty$. To confirm this equation algebraically, find $\lim_{x\to\infty} y$ by first dividing both the numerator and denominator by $e^{0.7x}$.

13.2: Absolute Extrema on a Closed Interval (Q. 1 – 13)

PROBLEMS 13.2

In Problems 1–14, find the absolute extrema of the given function on the given interval.

1. $f(x) = x^2 - 2x + 3$, $[0, 3]$
2. $f(x) = -3x^2 + 12x + 1$, $[1, 3]$
3. $f(x) = \frac{1}{3}x^3 + \frac{1}{2}x^2 - 2x + 1$, $[-1, 0]$
4. $f(x) = \frac{1}{4}x^4 - \frac{3}{2}x^2$, $[0, 1]$
5. $f(x) = x^3 - 5x^2 - 8x + 50$, $[0, 5]$
6. $f(x) = x^{2/3}$, $[-8, 8]$
7. $f(x) = (1/6)x^6 - (3/4)x^4 - 2x^2$, $[-1, 1]$
8. $f(x) = \frac{7}{3}x^3 + 2x^2 - 3x + 1$, $[0, 3]$
9. $f(x) = 3x^4 - x^6$, $[-1, 2]$
10. $f(x) = x^4 - 8x^3 + 22x^2 - 24x + 2$, $[0, 4]$
11. $f(x) = x^4 - 9x^2 + 2$, $[-1, 3]$
12. $f(x) = \frac{x}{x^2 - 1}$, $[2, 3]$
13. $f(x) = (x - 1)^{2/3}$, $[-26, 28]$
14. $f(x) = 0.2x^3 - 3.6x^2 + 2x + 1$, $[-1, 2]$
15. Consider the function

$$f(x) = x^4 + 8x^3 + 21x^2 + 20x + 9$$

over the interval $[-4, 9]$.

(a) Determine the value(s) (rounded to two decimal places) of x at which f attains a minimum value.
(b) What is the minimum value (rounded to two decimal places) of f?
(c) Determine the value(s) of x at which f attains a maximum value.
(d) What is the maximum value of f?

13.6: Applied Maxima and Minima (Q. 1—23, 27—37, 41)

PROBLEMS 13.6

In this set of problems, unless otherwise specified, p is price per unit (in dollars) and q is output per unit of time. Fixed costs refer to costs that remain constant at all levels of production during a given time period. (An example is rent.)

1. Find two numbers whose sum is 96 and whose product is as big as possible.

2. Find two nonnegative numbers whose sum is 20 and for which the product of twice one number and the square of the other number will be a maximum.

3. Fencing A company has set aside \$9000 to fence in a rectangular portion of land adjacent to a stream by using the stream for one side of the enclosed area. The cost of the fencing parallel to the stream is \$15 per foot, installed, and the fencing for the remaining two sides costs \$9 per foot, installed. Find the dimensions of the maximum enclosed area.

4. Fencing The owner of the Laurel Nursery Garden Center wants to fence in 1400 ft^2 of land in a rectangular plot to be used for different types of shrubs. The plot is to be divided into six equal plots with five fences parallel to the same pair of sides, as shown in Figure 13.65. What is the least number of feet of fence needed?

FIGURE 13.65

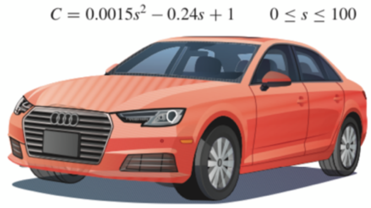
5. Average Cost A manufacturer finds that the total cost, c, of producing a product is given by the cost function

$$c = 0.05q^2 + 5q + 500$$

At what level of output will average cost per unit be a minimum?

6. Automobile Expense The cost per hour (in dollars) of operating an automobile is given by

$$C = 0.0015s^2 - 0.24s + 1 \qquad 0 \le s \le 100$$



where s is the speed in kilometers per hour. At what speed is the cost per hour a minimum?

7. Revenue The demand equation for a monopolist's product is

$$p = -5q + 30$$

At what price will revenue be maximized?

8. Revenue Suppose that the demand function for a monopolist's product is of the form

$$q = Ae^{-Bp}$$

for positive constants A and B. In terms of A and B, find the value of p for which maximum revenue is obtained. Can you explain why your answer does not depend on A?

9. Weight Gain A group of biologists studied the nutritional effects on rats that were fed a diet containing 10% protein.[12] The protein consisted of yeast and cottonseed flour. By varying the percent, p, of yeast in the protein mix, the group found that the (average) weight gain (in grams) of a rat over a period of time was

$$f(p) = 170 - p - \frac{1600}{p+15} \qquad 0 \le p \le 100$$

Find **(a)** the maximum weight gain and **(b)** the minimum weight gain.

10. Drug Dose The severity of the reaction of the human body to an initial dose, D, of a drug is given by[13]

$$R = f(D) = D^2\left(\frac{C}{2} - \frac{D}{3}\right)$$

where the constant C denotes the maximum amount of the drug that may be given. Show that R has a maximum *rate of change* when $D = C/2$.

11. Profit For a monopolist's product, the demand function is

$$p = 75 - 0.05q$$

and the cost function is

$$c = 500 + 40q$$

At what level of output will profit be maximized? At what price does this occur, and what is the profit?

12. Profit For a monopolist, the cost per unit of producing a product is \$3, and the demand equation is

$$p = \frac{10}{\sqrt{q}}$$

What price will give the greatest profit?

13. Profit For a monopolist's product, the demand equation is

$$p = 42 - 4q$$

and the average-cost function is

$$\bar{c} = 2 + \frac{80}{q}$$

Find the profit-maximizing price.

14. Profit For a monopolist's product, the demand function is

$$p = \frac{50}{\sqrt{q}}$$

and the average-cost function is

$$\bar{c} = \frac{1}{4} + \frac{2500}{q}$$

Find the profit-maximizing price and output.

15. Profit A manufacturer can produce at most 120 units of a certain product each year. The demand equation for the product is

$$p = q^2 - 100q + 3200$$

and the manufacturer's average-cost function is

$$\bar{c} = \frac{2}{3}q^2 - 40q + \frac{10{,}000}{q}$$

Determine the profit-maximizing output q and the corresponding maximum profit.

[12] Adapted from R. Bressani, "The Use of Yeast in Human Foods," in *Single-Cell Protein*, eds. R. I. Mateles and S. R. Tannenbaum (Cambridge, MA: MIT Press, 1968).

[13] R. M. Thrall, J. A. Mortimer. K. R. Rebman, and R. F. Baum, eds., *Some Mathematical Models in Biology*, rev. ed., Report No. 40241-R-7. Prepared at University of Michigan, 1967.

maximum revenue, and what will this revenue be?

16. Cost A manufacturer has determined that, for a certain product, the average cost (in dollars per unit) is given by

$$\bar{c} = 2q^2 - 48q + 210 + \frac{200}{q}$$

where $2 \leq q \leq 7$.
(a) At what level within the interval $[2, 7]$ should production be fixed in order to minimize total cost? What is the minimum total cost?
(b) If production were required to lie in the interval $[3, 7]$, what value of q would minimize total cost?

17. Profit For XYZ Manufacturing Co., total fixed costs are \$1200, material and labor costs combined are \$2 per unit, and the demand equation is

$$p = \frac{100}{\sqrt{q}}$$

What level of output will maximize profit? Show that this occurs when marginal revenue is equal to marginal cost. What is the price at profit maximization?

18. Revenue A real-estate firm owns 100 garden-type apartments. At \$400 per month, each apartment can be rented. However, for each \$10-per-month increase, there will be two vacancies with no possibility of filling them. What rent per apartment will maximize monthly revenue?

19. Revenue A TV cable company has 6400 subscribers who are each paying \$24 per month. It can get 160 more subscribers for each \$0.50 decrease in the monthly fee. What rate will yield maximum revenue, and what will this revenue be?

20. Profit A manufacturer of a product finds that, for the first 600 units that are produced and sold, the profit is \$40 per unit. The profit on each of the units beyond 600 is decreased by \$0.05 times the number of additional units produced. For example, the total profit when 602 units are produced and sold is $600(40) + 2(39.90)$. What level of output will maximize profit?

21. Container Design A container manufacturer is designing a rectangular box, open at the top and with a square base, that is to have a volume of 13.5 ft^3. If the box is to require the least amount of material, what must be its dimensions?

22. Container Design An open-top box with a square base is to be constructed from 192 ft^2 of material. What should be the dimensions of the box if the volume is to be a maximum? What is the maximum volume?

23. Container Design An open box is to be made by cutting equal squares from each corner of a L-inch-square piece of cardboard and then folding up the sides. Find the length of the side of the square (in terms of L) that must be cut out if the volume of the box is to be maximized. What is the maximum volume? (See Figure 13.66.)

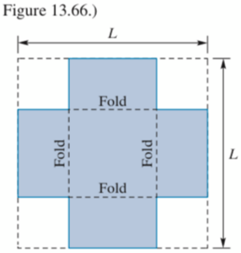


FIGURE 13.66

29. Economic Lot Size A manufacturer has to produce 3000 units annually of a product that is sold at a uniform rate during the year. The production cost of each unit is \$12, and carrying costs (insurance, interest, storage, etcetera) are estimated to be 19.2% of the value of average inventory. Setup costs per production run are \$54. Find the economic lot size.

30. Profit For a monopolist's product, the cost function is

$$c = 0.004q^3 + 20q + 5000$$

and the demand function is

$$p = 450 - 4q$$

Find the profit-maximizing output.

31. Workshop Attendance Imperial Educational Services (I.E.S.) is considering offering a workshop in resource allocation to key personnel at Acme Corp. To make the offering economically feasible, I.E.S. says that at least 40 persons must attend at a cost of \$200 each. Moreover, I.E.S. will agree to reduce the charge for *everybody* by \$2.50, for each person over the committed 40, who attends. How many people should be in the group for I.E.S. to maximize revenue? Assume that the maximum allowable number in the group is 70.

32. Cost of Leasing Motor The Kiddie Toy Company plans to lease an electric motor that will be used 80,000 horsepower-hours per year in manufacturing. One horsepower-hour is the work done in 1 hour by a 1-horsepower motor. The annual cost to lease a suitable motor is \$200, plus \$0.40 per horsepower. The cost per horsepower-hour of operating the motor is \$0.008/N, where N is the horsepower. What size motor, in horsepower, should be leased in order to minimize cost?

33. Transportation Cost The cost of operating a truck on a thruway (excluding the salary of the driver) is

$$0.165 + \frac{s}{200}$$

dollars per mile, where s is the (steady) speed of the truck in miles per hour. The truck driver's salary is \$18 per hour. At what speed should the truck driver operate the truck to make a 700-mile trip most economical?

34. Cost For a manufacturer, the cost of making a part is \$30 per unit for labor and \$10 per unit for materials; overhead is fixed at \$20,000 per week. If more than 5000 units are made each week, labor is \$45 per unit for those units in excess of 5000. At what level of production will average cost per unit be a minimum?

35. Profit Ms. Jones owns a small insurance agency that sells policies for a large insurance company. For each policy sold, Ms. Jones, who does not sell policies herself, is paid a commission of \$50 by the insurance company. From previous experience, Ms. Jones has determined that, when she employs m salespeople,

$$q = m^3 - 15m^2 + 92m$$

policies can be sold per week. She pays each of the m salespeople a salary of \$1000 per week, and her weekly fixed cost is \$3000. Current office facilities can accommodate at most eight salespeople. Determine the number of salespeople that Ms. Jones should hire to maximize her weekly profit. What is the corresponding maximum profit?

36. Profit A manufacturing company sells high-quality jackets through a chain of specialty shops. The demand equation for these jackets is

$$p = 1000 - 50q$$

where p is the selling price (in dollars per jacket) and q is the demand (in thousands of jackets). If this company's marginal-cost function is given by

$$\frac{dc}{dq} = \frac{1000}{q+5}$$

show that there is a maximum profit, and determine the number of jackets that must be sold to obtain this maximum profit.

37. Chemical Production Each day, a firm makes x tons of chemical A ($x \le 4$) and

$$y = \frac{24 - 6x}{5 - x}$$

tons of chemical B. The profit on chemical A is \$2000 per ton, and on B it is \$1000 per ton. How much of chemical A should be produced per day to maximize profit? Answer the same question if the profit on A is P per ton and that on B is $P/2$ per ton.

38. Rate of Return To erect an office building, fixed costs are \$1.44 million and include land, architect's fees, a basement, a foundation, and so on. If x floors are constructed, the cost (excluding fixed costs) is

$$c = 10x[120{,}000 + 3000(x - 1)]$$

The revenue per month is \$60,000 per floor. How many floors will yield a maximum rate of return on investment? (Rate of return = total revenue/total cost.)

39. Gait and Power Output of an Animal In a model by Smith,[14] the power output of an animal at a given speed as a function of its movement or *gait*, j, is found to be

$$P(j) = Aj\frac{L^4}{V} + B\frac{V^3L^2}{1+j}$$

where A and B are constants, j is a measure of the "jumpiness" of the gait, L is a constant representing linear dimension, and V is a constant forward speed.

[14] J. M. Smith, *Mathematical Ideas in Biology* (London: Cambridge University Press, 1968).

13.4: Second Derivative Test (Q. 1—13)

PROBLEMS 13.4

In Problems 1–14, test for relative maxima and minima. Use the second-derivative test, if possible. In Problems 1–4, state whether the relative extrema are also absolute extrema.

1. $y = x^2 - 5x + 6$

2. $y = 3x^2 + 12x + 14$

3. $y = -4x^2 + 2x - 8$

4. $y = -5x^2 + 11x - 7$

5. $y = \frac{1}{3}x^3 + 2x^2 - 5x + 1$

6. $y = x^3 - 12x + 1$

7. $y = 2x^3 - 3x^2 - 36x + 17$

8. $y = x^4 - 2x^2 + 4$

9. $y = 3 + 5x^4$

10. $y = -2x^7$

11. $y = 81x^5 - 5x$

12. $y = 15x^3 + x^2 - 15x + 2$

13. $y = (x^2 + 7x + 10)^2$

14. $y = 2x^3 - 9x^2 - 60x + 42$